11

The circle and its properties

11.1 Introduction

A **circle** is a plain figure enclosed by a curved line, every point on which is equidistant from a point within, called the **centre**.

11.2 Properties of circles

(i) The distance from the centre to the curve is called the **radius**, r, of the circle (see OP in Fig. 11.1).

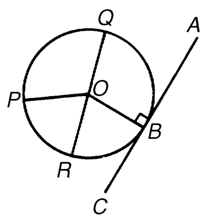

Figure 11.1

(ii) The boundary of a circle is called the **circumference**, c.

(iii) Any straight line passing through the centre and touching the circumference at each end is called the **diameter**, d (see QR in Fig. 11.1). Thus $\boldsymbol{d = 2r}$.

(iv) The ratio $\dfrac{\text{circumference}}{\text{diameter}} = $ a constant for any circle.
This constant is denoted by the Greek letter π (pronounced 'pie'), where $\pi = 3.14159$, correct to 5 decimal places.
Hence $c/d = \pi$ or $\boldsymbol{c = \pi d}$ or $\boldsymbol{c = 2\pi r}$.

(v) A **semicircle** is one half of the whole circle.

(vi) A **quadrant** is one quarter of a whole circle.

(vii) A **tangent** to a circle is a straight line which meets the circle in one point only and does not cut the circle when produced. AC in Fig. 11.1 is a tangent to the circle since it touches the curve at point B only. If radius OB is drawn, then angle ABO is a right angle.

(viii) A **sector** of a circle is the part of a circle between radii (for example, the portion OXY of Fig. 11.2 is a sector). If a sector is less than a semicircle it is called a **minor sector**, if greater than a semicircle it is called a **major sector**.

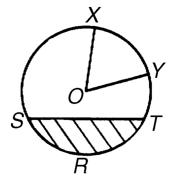

Figure 11.2

(ix) A **chord** of a circle is any straight line which divides the circle into two parts and is terminated at each end by the circumference. ST, in Fig. 11.2 is a chord.

(x) A **segment** is the name given to the parts into which a circle is divided by a chord. If the segment is less than a semicircle it is called a **minor segment** (see shaded area in Fig. 11.2). If the segment is greater than a semicircle it is called a **major segment** (see the unshaded area in Fig. 11.2).

(xi) An **arc** is a portion of the circumference of a circle. The distance SRT in Fig. 11.2 is called a **minor arc** and the distance $SXYT$ is called a **major arc**.

(xii) The angle at the centre of a circle, subtended by an arc, is double the angle at the circumference subtended by the same arc. With reference to Fig. 11.3, **Angle** $\boldsymbol{AOC = 2 \times}$ **angle** $\boldsymbol{ABC}$.

(xiii) The angle in a semicircle is a right angle (see angle BQP in Fig. 11.3).

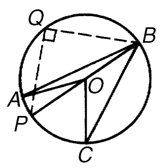

Figure 11.3

Problem 1. If the diameter of a circle is 75 mm, find its circumference.

Circumference, $c = \pi \times \text{diameter} = \pi d$
$= \pi(75) = \mathbf{235.6\,mm}$.

Problem 2. In Fig. 11.4, AB is a tangent to the circle at B. If the circle radius is 40 mm and $AB = 150$ mm, calculate the length AO.

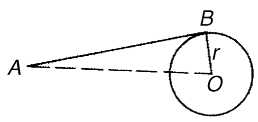

Figure 11.4

A tangent to a circle is at right angles to a radius drawn from the point of contact, i.e. $ABO = 90°$. Hence, using Pythagoras' theorem:

$$AO^2 = AB^2 + OB^2$$
$$AO = \sqrt{(AB^2 + OB^2)} = \sqrt{[(150)^2 + (40)^2]}$$
$$= \mathbf{155.2\,mm}$$

Now try the following exercise.

Exercise 48 Further problems on properties of circles

1. If the diameter of a circle is 82.6 mm, calculate the circumference of the circle. [259.5 mm]
2. Find the diameter of a circle whose perimeter is 149.8 cm. [47.68 cm]

11.3 Arc length and area of a sector

One **radian** is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius. With reference to Fig. 11.5, for arc length s,

$$\theta \text{ radians} = s/r \text{ or } \textbf{arc length, } \boxed{s = r\theta} \quad (1)$$

where θ is in radians.

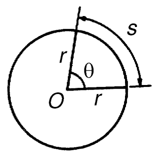

Figure 11.5

When $s =$ whole circumference ($= 2\pi r$) then $\theta = s/r = 2\pi r/r = 2\pi$.

i.e. $2\pi \text{ rad} = 360°$ or $\boxed{\pi \textbf{ rad} = \mathbf{180°}}$

Thus 1 rad $= 180°/\pi = 57.30°$, correct to 2 decimal places.

Since π rad $= 180°$, then $\pi/2 = 90°, \pi/3 = 60°, \pi/4 = 45°$, and so on.

$$\textbf{Area of a sector} = \frac{\theta}{360}(\pi r^2) \quad \text{when } \theta \text{ is in degrees}$$
$$= \frac{\theta}{2\pi}(\pi r^2) = \frac{1}{2}r^2\theta \quad (2)$$
$$\text{when } \theta \text{ is in radians}$$

Problem 3. Convert to radians: (a) 125° (b) 69°47′.

(a) Since $180° = \pi$ rad then $1° = \pi/180$ rad, therefore

$$125° = 125\left(\frac{\pi}{180}\right)^c = \mathbf{2.182\ rad}$$

(Note that ^{c} means 'circular measure' and indicates radian measure.)

(b) $69°47' = 69\frac{47°}{60} = 69.783°$

$$69.783° = 69.783\left(\frac{\pi}{180}\right)^c = \mathbf{1.218\ rad}$$

Problem 4. Convert to degrees and minutes: (a) 0.749 rad (b) $3\pi/4$ rad.

(a) Since π rad $= 180°$ then 1 rad $= 180°/\pi$, therefore

$$0.749 = 0.749\left(\frac{180}{\pi}\right)^\circ = 42.915°$$

$0.915° = (0.915 \times 60)' = 55'$, correct to the nearest minute, hence

$$\mathbf{0.749\ rad = 42°55'}$$

(b) Since $1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$ then

$$\frac{3\pi}{4} \text{ rad} = \frac{3\pi}{4}\left(\frac{180}{\pi}\right)^\circ = \frac{3}{4}(180)^\circ = \mathbf{135°}.$$

> Problem 5. Express in radians, in terms of π, (a) 150° (b) 270° (c) 37.5°.

Since $180° = \pi$ rad then $1° = 180/\pi$, hence

(a) $150° = 150\left(\frac{\pi}{180}\right) \text{rad} = \mathbf{\frac{5\pi}{6}\ rad}$

(b) $270° = 270\left(\frac{\pi}{180}\right) \text{rad} = \mathbf{\frac{3\pi}{2}\ rad}$

(c) $37.5° = 37.5\left(\frac{\pi}{180}\right) \text{rad} = \frac{75\pi}{360} \text{rad} = \mathbf{\frac{5\pi}{24}\ rad}$

Now try the following exercise.

Exercise 49 Further problems on radians and degrees

1. Convert to radians in terms of π: (a) 30° (b) 75° (c) 225°. $\left[\text{(a) } \frac{\pi}{6} \text{ (b) } \frac{5\pi}{12} \text{ (c) } \frac{5\pi}{4}\right]$

2. Convert to radians: (a) 48° (b) 84°51′ (c) 232°15′.
[(a) 0.838 (b) 1.481 (c) 4.054]

3. Convert to degrees: (a) $\frac{5\pi}{6}$ rad (b) $\frac{4\pi}{9}$ rad (c) $\frac{7\pi}{12}$ rad. [(a) 150° (b) 80° (c) 105°]

4. Convert to degrees and minutes: (a) 0.0125 rad (b) 2.69 rad (c) 7.241 rad.
[(a) 0°43′ (b) 154°8′ (c) 414°53′]

11.4 Worked problems on arc length and sector of a circle

> Problem 6. Find the length of arc of a circle of radius 5.5 cm when the angle subtended at the centre is 1.20 rad.

From equation (1), length of arc, $s = r\theta$, where θ is in radians, hence

$$s = (5.5)(1.20) = \mathbf{6.60\ cm}$$

> Problem 7. Determine the diameter and circumference of a circle if an arc of length 4.75 cm subtends an angle of 0.91 rad.

Since $s = r\theta$ then $r = \frac{s}{\theta} = \frac{4.75}{0.91} = 5.22$ cm

Diameter $= 2 \times$ radius $= 2 \times 5.22 = \mathbf{10.44\ cm}$
Circumference, $c = \pi d = \pi(10.44) = \mathbf{32.80\ cm}$

> Problem 8. If an angle of 125° is subtended by an arc of a circle of radius 8.4 cm, find the length of (a) the minor arc, and (b) the major arc, correct to 3 significant figures.

(a) Since $180° = \pi$ rad then $1° = \left(\frac{\pi}{180}\right)$ rad and $125° = 125\left(\frac{\pi}{180}\right)$ rad.

Length of minor arc,

$$s = r\theta = (8.4)(125)\left(\frac{\pi}{180}\right) = \mathbf{18.3\ cm},$$

correct to 3 significant figures.

(b) Length of major arc

$= $ (circumference − minor arc)
$= 2\pi(8.4) - 18.3 = 34.5$ cm,
correct to 3 significant figures.

(Alternatively, major arc $= r\theta$
$= 8.4(360 - 125)(\pi/180) = \mathbf{34.5\ cm}$.)

> Problem 9. Determine the angle, in degrees and minutes, subtended at the centre of a circle of diameter 42 mm by an arc of length 36 mm. Calculate also the area of the minor sector formed.

Since length of arc, $s = r\theta$ then $\theta = s/r$

$$\text{Radius, } r = \frac{\text{diameter}}{2} = \frac{42}{2} = 21 \text{ mm}$$

$$\text{hence } \theta = \frac{s}{r} = \frac{36}{21} = 1.7143 \text{ rad}$$

$1.7143 \text{ rad} = 1.7143 \times (180/\pi)° = 98.22° = \mathbf{98°13'}$ = angle subtended at centre of circle.

From equation (2), **area of sector**
$= \frac{1}{2}r^2\theta = \frac{1}{2}(21)^2(1.7143) = \mathbf{378\,mm^2}$.

> Problem 10. A football stadium floodlight can spread its illumination over an angle of 45° to a distance of 55 m. Determine the maximum area that is floodlit.

Floodlit area = area of sector

$$= \frac{1}{2}r^2\theta = \frac{1}{2}(55)^2\left(45 \times \frac{\pi}{180}\right),$$
from equation (2)

$$= \mathbf{1188\,m^2}$$

> Problem 11. An automatic garden spray produces a spray to a distance of 1.8 m and revolves through an angle α which may be varied. If the desired spray catchment area is to be 2.5 m^2, to what should angle α be set, correct to the nearest degree.

Area of sector $= \frac{1}{2}r^2\theta$, hence $2.5 = \frac{1}{2}(1.8)^2\alpha$

from which, $\alpha = \dfrac{2.5 \times 2}{1.8^2} = 1.5432$ rad

$$1.5432\text{ rad} = \left(1.5432 \times \frac{180°}{\pi}\right) = 88.42°$$

Hence **angle $\boldsymbol{\alpha} = 88°$**, correct to the nearest degree.

Now try the following exercise.

Exercise 50 Further problems on arc length and sector of a circle

1. Find the length of an arc of a circle of radius 8.32 cm when the angle subtended at the centre is 2.14 rad. Calculate also the area of the minor sector formed. [17.80 cm, 74.07 cm^2]
2. If the angle subtended at the centre of a circle of diameter 82 mm is 1.46 rad, find the lengths of the (a) minor arc (b) major arc. [(a) 59.86 mm (b) 197.8 mm]
3. A pendulum of length 1.5 m swings through an angle of 10° in a single swing. Find, in centimetres, the length of the arc traced by the pendulum bob. [26.2 cm]
4. Determine the length of the radius and circumference of a circle if an arc length of 32.6 cm subtends an angle of 3.76 rad. [8.67 cm, 54.48 cm]
5. Determine the angle of lap, in degrees and minutes, if 180 mm of a belt drive are in contact with a pulley of diameter 250 mm. [82°30′]
6. Determine the number of complete revolutions a motorcycle wheel will make in travelling 2 km, if the wheel's diameter is 85.1 cm. [748]
7. The floodlights at a sports ground spread its illumination over an angle of 40° to a distance of 48 m. Determine (a) the angle in radians, and (b) the maximum area that is floodlit. [(a) 0.698 rad (b) 804.2 m^2]
8. Determine (a) the shaded area in Fig. 11.6 (b) the percentage of the whole sector that the area of the shaded area represents. [(a) 396 mm^2 (b) 42.24%]

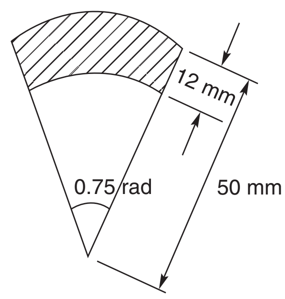

Figure 11.6

11.5 The equation of a circle

The simplest equation of a circle, centre at the origin, radius r, is given by:

$$x^2 + y^2 = r^2$$

For example, Fig. 11.7 shows a circle $x^2 + y^2 = 9$. More generally, the equation of a circle, centre (a, b), radius r, is given by:

$$(x - a)^2 + (y - b)^2 = r^2 \qquad (1)$$

Figure 11.8 shows a circle $(x - 2)^2 + (y - 3)^2 = 4$. The general equation of a circle is:

$$x^2 + y^2 + 2ex + 2fy + c = 0 \qquad (2)$$

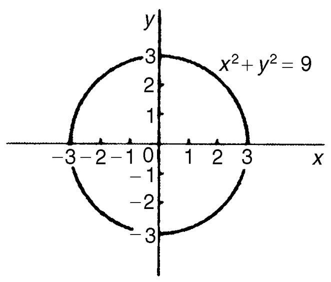

Figure 11.7

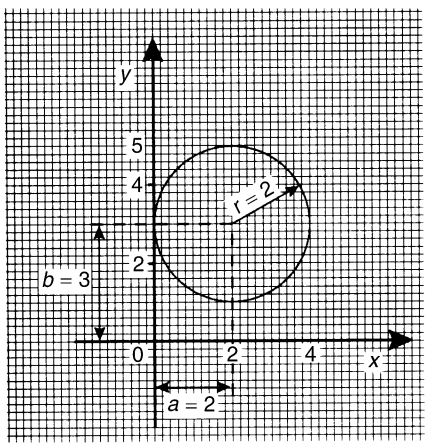

Figure 11.8

Multiplying out the bracketed terms in equation (1) gives:

$$x^2 - 2ax + a^2 + y^2 - 2by + b^2 = r^2$$

Comparing this with equation (2) gives:

$$2e = -2a, \text{ i.e. } \boldsymbol{a = -\frac{2e}{2}}$$

and $2f = -2b$, i.e. $\boldsymbol{b = -\frac{2f}{2}}$

and $c = a^2 + b^2 - r^2$,

i.e., $\boldsymbol{r = \sqrt{(a^2 + b^2 - c)}}$

Thus, for example, the equation

$$x^2 + y^2 - 4x - 6y + 9 = 0$$

represents a circle with centre $a = -\left(\frac{-4}{2}\right)$, $b = -\left(\frac{-6}{2}\right)$, i.e., at (2, 3) and radius $r = \sqrt{(2^2 + 3^2 - 9)} = 2$.

Hence $x^2 + y^2 - 4x - 6y + 9 = 0$ is the circle shown in Fig. 11.8 (which may be checked by multiplying out the brackets in the equation

$$(x - 2)^2 + (y - 3)^2 = 4$$

Problem 12. Determine (a) the radius, and (b) the co-ordinates of the centre of the circle given by the equation: $x^2 + y^2 + 8x - 2y + 8 = 0$.

$x^2 + y^2 + 8x - 2y + 8 = 0$ is of the form shown in equation (2),

where $a = -\left(\frac{8}{2}\right) = -4, b = -\left(\frac{-2}{2}\right) = 1$

and $r = \sqrt{[(-4)^2 + (1)^2 - 8]} = \sqrt{9} = 3$

Hence $x^2 + y^2 + 8x - 2y + 8 = 0$ represents a circle **centre (−4, 1)** and **radius 3**, as shown in Fig. 11.9.

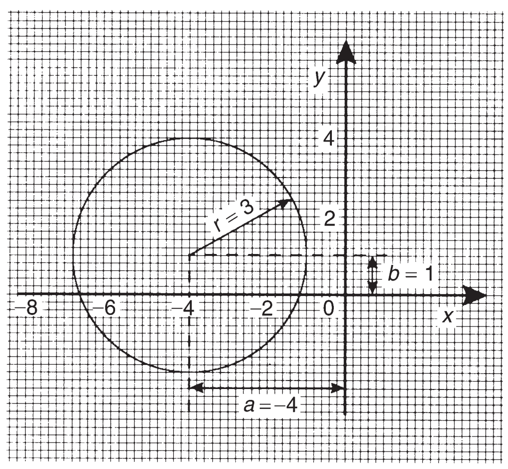

Figure 11.9

Problem 13. Sketch the circle given by the equation: $x^2 + y^2 - 4x + 6y - 3 = 0$.

The equation of a circle, centre (a, b), radius r is given by:

$$(x - a)^2 + (y - b)^2 = r^2$$

The general equation of a circle is

$$x^2 + y^2 + 2ex + 2fy + c = 0.$$

From above $a = -\frac{2e}{2}$, $b = -\frac{2f}{2}$ and

$$r = \sqrt{(a^2 + b^2 - c)}.$$

Hence if $x^2 + y^2 - 4x + 6y - 3 = 0$

then $a = -\left(\frac{-4}{2}\right) = 2, \quad b = -\left(\frac{6}{2}\right) = -3$

and $r = \sqrt{[(2)^2 + (-3)^2 - (-3)]}$
$= \sqrt{16} = 4$

Thus **the circle has centre (2, −3) and radius 4**, as shown in Fig. 11.10.

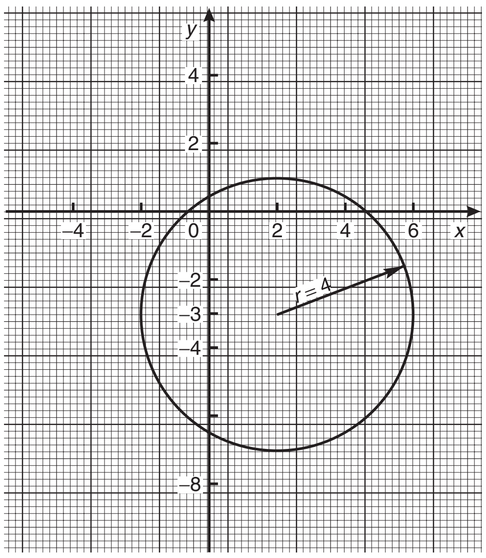

Figure 11.10

Now try the following exercise.

Exercise 51 Further problems on the equation of a circle

1. Determine (a) the radius, and (b) the coordinates of the centre of the circle given by the equation $x^2 + y^2 + 6x - 2y - 26 = 0$.
[(a) 6 (b) (−3, 1)]
2. Sketch the circle given by the equation $x^2 + y^2 - 6x + 4y - 3 = 0$.
[Centre at (3, −2), radius 4]
3. Sketch the curve $x^2 + (y - 1)^2 - 25 = 0$.
[Circle, centre (0, 1), radius 5]
4. Sketch the curve $x = 6\sqrt{\left[1 - (y/6)^2\right]}$.
[Circle, centre (0, 0), radius 6]

11.6 Linear and angular velocity

Linear velocity

Linear velocity v is defined as the rate of change of linear displacement s with respect to time t. For motion in a straight line:

$$\text{linear velocity} = \frac{\text{change of displacement}}{\text{change of time}}$$

i.e.
$$\boldsymbol{v = \frac{s}{t}} \qquad (1)$$

The unit of linear velocity is metres per second (m/s).

Angular velocity

The speed of revolution of a wheel or a shaft is usually measured in revolutions per minute or revolutions per second but these units do not form part of a coherent system of units. The basis in SI units is the angle turned through in one second.

Angular velocity is defined as the rate of change of angular displacement θ, with respect to time t. For an object rotating about a fixed axis at a constant speed:

$$\text{angular velocity} = \frac{\text{angle turned through}}{\text{time taken}}$$

i.e.
$$\boldsymbol{\omega = \frac{\theta}{t}} \qquad (2)$$

The unit of angular velocity is radians per second (rad/s). An object rotating at a constant speed of n revolutions per second subtends an angle of $2\pi n$ radians in one second, i.e., its angular velocity ω is given by:

$$\boldsymbol{\omega = 2\pi n \text{ rad/s}} \qquad (3)$$

From equation (1) on page 100, $s = r\theta$ and from equation (2), $\theta = \omega t$

hence $s = r(\omega t)$

from which $\frac{s}{t} = \omega r$

However, from equation (1) $v = \frac{s}{t}$

hence
$$\boldsymbol{v = \omega r} \qquad (4)$$

Equation (4) gives the relationship between linear velocity v and angular velocity ω.

> Problem 14. A wheel of diameter 540 mm is rotating at $\frac{1500}{\pi}$ rev/min. Calculate the angular velocity of the wheel and the linear velocity of a point on the rim of the wheel.

From equation (3), angular velocity $\omega = 2\pi n$ where n is the speed of revolution in rev/s. Since in this case $n = \frac{1500}{\pi}$ rev/min $= \frac{1500}{60\pi}$ = rev/s, then

$$\textbf{angular velocity } \boldsymbol{\omega} = 2\pi\left(\frac{1500}{60\pi}\right) = \mathbf{50\,rad/s}$$

The linear velocity of a point on the rim, $v = \omega r$, where r is the radius of the wheel, i.e. $\frac{540}{2}$ mm $= \frac{0.54}{2}$ m $= 0.27$ m.

Thus **linear velocity** $\quad \boldsymbol{v} = \omega r = (50)(0.27)$
$= \mathbf{13.5\,m/s}$

> Problem 15. A car is travelling at 64.8 km/h and has wheels of diameter 600 mm.
> (a) Find the angular velocity of the wheels in both rad/s and rev/min.
> (b) If the speed remains constant for 1.44 km, determine the number of revolutions made by the wheel, assuming no slipping occurs.

(a) Linear velocity $v = 64.8$ km/h

$$= 64.8\,\frac{\text{km}}{\text{h}} \times 1000\,\frac{\text{m}}{\text{km}} \times \frac{1}{3600}\,\frac{\text{h}}{\text{s}} = 18\,\text{m/s}.$$

The radius of a wheel $= \frac{600}{2} = 300$ mm $= 0.3$ m.

From equation (5), $v = \omega r$, from which,

$$\textbf{angular velocity } \boldsymbol{\omega} = \frac{v}{r} = \frac{18}{0.3} = \mathbf{60\,rad/s}$$

From equation (4), angular velocity, $\omega = 2\pi n$, where n is in rev/s.

Hence angular speed $n = \frac{\omega}{2\pi} = \frac{60}{2\pi}$ rev/s

$= 60 \times \frac{60}{2\pi}$ rev/min

$= \mathbf{573\,rev/min}$

(b) From equation (1), since $v = s/t$ then the time taken to travel 1.44 km, i.e., 1440 m at a constant speed of 18 m/s is given by:

$$\text{time } t = \frac{s}{v} = \frac{1440\,\text{m}}{18\,\text{m/s}} = 80\,\text{s}$$

Since a wheel is rotating at 573 rev/min, then in 80/60 minutes it makes

$$573\,\text{rev/min} \times \frac{80}{60}\,\text{min} = \mathbf{764\ revolutions}$$

Now try the following exercise.

> **Exercise 52 Further problems on linear and angular velocity**
>
> 1. A pulley driving a belt has a diameter of 300 mm and is turning at $2700/\pi$ revolutions per minute. Find the angular velocity of the pulley and the linear velocity of the belt assuming that no slip occurs. [$\omega = 90$ rad/s, $v = 13.5$ m/s]
> 2. A bicycle is travelling at 36 km/h and the diameter of the wheels of the bicycle is 500 mm. Determine the angular velocity of the wheels of the bicycle and the linear velocity of a point on the rim of one of the wheels. [$\omega = 40$ rad/s, $v = 10$ m/s]
> 3. A train is travelling at 108 km/h and has wheels of diameter 800 mm.
>   (a) Determine the angular velocity of the wheels in both rad/s and rev/min.
>   (b) If the speed remains constant for 2.70 km, determine the number of revolutions made by a wheel, assuming no slipping occurs.
>   [(a) 75 rad/s, 716.2 rev/min (b) 1074 revs]

11.7 Centripetal force

When an object moves in a circular path at constant speed, its direction of motion is continually changing and hence its velocity (which depends on both magnitude and direction) is also continually changing. Since acceleration is the (change in velocity)/(time taken), the object has an acceleration. Let the object be moving with a constant angular velocity of ω and a tangential velocity of magnitude v and let the change of velocity for a small change of angle of θ $(=\omega t)$ be V in Fig. 11.11. Then $v_2 - v_1 = V$. The vector diagram is shown in Fig. 11.11(b) and since the magnitudes of v_1 and v_2 are the same, i.e. v, the vector diagram is an isosceles triangle.

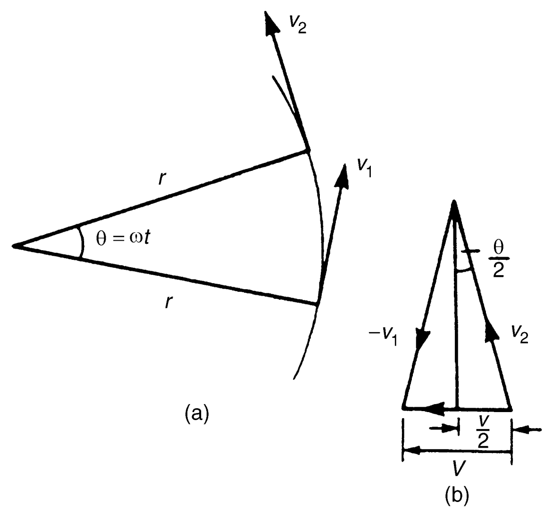

Figure 11.11

Bisecting the angle between v_2 and v_1 gives:

$$\sin\frac{\theta}{2} = \frac{V/2}{v_2} = \frac{V}{2v}$$

i.e. $V = 2v\sin\frac{\theta}{2}$ (1)

Since $\theta = \omega t$ then

$$t = \frac{\theta}{\omega} \qquad (2)$$

Dividing equation (1) by equation (2) gives:

$$\frac{V}{t} = \frac{2v\sin(\theta/2)}{(\theta/\omega)} = \frac{v\omega\sin(\theta/2)}{(\theta/2)}$$

For small angles $\frac{\sin(\theta/2)}{(\theta/2)} \approx 1$,

hence $\frac{V}{t} = \frac{\text{change of velocity}}{\text{change of time}}$
$= \text{acceleration } a = v\omega$

However, $\omega = \frac{v}{r}$ (from Section 11.6)

thus $v\omega = v \cdot \frac{v}{r} = \frac{v^2}{r}$

i.e. the acceleration a is $\frac{v^2}{r}$ and is towards the centre of the circle of motion (along V). It is called the **centripetal acceleration**. If the mass of the rotating object is m, then by Newton's second law, the **centripetal force is** $\boldsymbol{\frac{mv^2}{r}}$ and its direction is towards the centre of the circle of motion.

Problem 16. A vehicle of mass 750 kg travels around a bend of radius 150 m, at 50.4 km/h. Determine the centripetal force acting on the vehicle.

The centripetal force is given by $\frac{mv^2}{r}$ and its direction is towards the centre of the circle.

Mass $m = 750$ kg, $v = 50.4$ km/h

$$= \frac{50.4 \times 1000}{60 \times 60} \text{ m/s}$$
$$= 14 \text{ m/s}$$

and radius $r = 150$ m,

thus **centripetal force** $= \frac{750(14)^2}{150} =$ **980 N**.

Problem 17. An object is suspended by a thread 250 mm long and both object and thread move in a horizontal circle with a constant angular velocity of 2.0 rad/s. If the tension in the thread is 12.5 N, determine the mass of the object.

Centripetal force (i.e. tension in thread),

$$F = \frac{mv^2}{r} = 12.5 \text{ N}$$

Angular velocity $\omega = 2.0$ rad/s and
radius $r = 250$ mm $= 0.25$ m.

Since linear velocity $v = \omega r$, $v = (2.0)(0.25)$
$= 0.5$ m/s.

Since $F = \dfrac{mv^2}{r}$, then mass $m = \dfrac{Fr}{v^2}$,

i.e. **mass of object,** $\boldsymbol{m} = \dfrac{(12.5)(0.25)}{0.5^2} = \mathbf{12.5\ kg}$

Problem 18. An aircraft is turning at constant altitude, the turn following the arc of a circle of radius 1.5 km. If the maximum allowable acceleration of the aircraft is $2.5g$, determine the maximum speed of the turn in km/h. Take g as $9.8\ m/s^2$.

The acceleration of an object turning in a circle is $\dfrac{v^2}{r}$. Thus, to determine the maximum speed of turn, $\dfrac{v^2}{r} = 2.5g$, from which,

$$\textbf{velocity, } \boldsymbol{v} = \sqrt{(2.5gr)} = \sqrt{(2.5)(9.8)(1500)} = \sqrt{36750} = 191.7\ m/s$$

and $191.7\ m/s = 191.7 \times \dfrac{60 \times 60}{1000}\ km/h = \mathbf{690\ km/h}$

Now try the following exercise.

Exercise 53 Further problems on centripetal force

1. Calculate the tension in a string when it is used to whirl a stone of mass 200 g round in a horizontal circle of radius 90 cm with a constant speed of 3 m/s. [2 N]

2. Calculate the centripetal force acting on a vehicle of mass 1 tonne when travelling around a bend of radius 125 m at 40 km/h. If this force should not exceed 750 N, determine the reduction in speed of the vehicle to meet this requirement. [988 N, 5.1 km/h]

3. A speed-boat negotiates an S-bend consisting of two circular arcs of radii 100 m and 150 m. If the speed of the boat is constant at 34 km/h, determine the change in acceleration when leaving one arc and entering the other. [$1.49\ m/s^2$]