

Part 1 Number and Algebra

1

Revision of fractions, decimals and percentages

1.1 Fractions

When 2 is divided by 3, it may be written as $\frac{2}{3}$ or $2/3$. $\frac{2}{3}$ is called a **fraction**. The number above the line, i.e. 2, is called the **numerator** and the number below the line, i.e. 3, is called the **denominator**.

When the value of the numerator is less than the value of the denominator, the fraction is called a **proper fraction**; thus $\frac{2}{3}$ is a proper fraction. When the value of the numerator is greater than the denominator, the fraction is called an **improper fraction**. Thus $\frac{7}{3}$ is an improper fraction and can also be expressed as a **mixed number**, that is, an integer and a proper fraction. Thus the improper fraction $\frac{7}{3}$ is equal to the mixed number $2\frac{1}{3}$.

When a fraction is simplified by dividing the numerator and denominator by the same number, the process is called **cancelling**. Cancelling by 0 is not permissible.

Problem 1. Simplify $\frac{1}{3} + \frac{2}{7}$

The lowest common multiple (i.e. LCM) of the two denominators is 3×7 , i.e. 21

Expressing each fraction so that their denominators are 21, gives:

$$\begin{aligned}\frac{1}{3} + \frac{2}{7} &= \frac{1}{3} \times \frac{7}{7} + \frac{2}{7} \times \frac{3}{3} = \frac{7}{21} + \frac{6}{21} \\ &= \frac{7+6}{21} = \frac{13}{21}\end{aligned}$$

Alternatively:

$$\frac{1}{3} + \frac{2}{7} = \frac{\begin{array}{c} \text{Step (2)} \\ \downarrow \\ (7 \times 1) \end{array} + \begin{array}{c} \text{Step (3)} \\ \downarrow \\ (3 \times 2) \end{array}}{\begin{array}{c} 21 \\ \uparrow \\ \text{Step (1)} \end{array}}$$

Step 1: the LCM of the two denominators;

Step 2: for the fraction $\frac{1}{3}$, 3 into 21 goes 7 times, $7 \times$ the numerator is 7×1 ;

Step 3: for the fraction $\frac{2}{7}$, 7 into 21 goes 3 times, $3 \times$ the numerator is 3×2 .

Thus $\frac{1}{3} + \frac{2}{7} = \frac{7+6}{21} = \frac{13}{21}$ as obtained previously.

Problem 2. Find the value of $3\frac{2}{3} - 2\frac{1}{6}$

One method is to split the mixed numbers into integers and their fractional parts. Then

$$\begin{aligned}3\frac{2}{3} - 2\frac{1}{6} &= \left(3 + \frac{2}{3}\right) - \left(2 + \frac{1}{6}\right) \\ &= 3 + \frac{2}{3} - 2 - \frac{1}{6} \\ &= 1 + \frac{4}{6} - \frac{1}{6} = 1\frac{3}{6} = 1\frac{1}{2}\end{aligned}$$

Another method is to express the mixed numbers as improper fractions.

$$\text{Since } 3 = \frac{9}{3}, \text{ then } 3\frac{2}{3} = \frac{9}{3} + \frac{2}{3} = \frac{11}{3}$$

$$\text{Similarly, } 2\frac{1}{6} = \frac{12}{6} + \frac{1}{6} = \frac{13}{6}$$

$$\text{Thus } 3\frac{2}{3} - 2\frac{1}{6} = \frac{11}{3} - \frac{13}{6} = \frac{22}{6} - \frac{13}{6} = \frac{9}{6} = \frac{3}{2}$$

as obtained previously.

Problem 3. Determine the value of

$$4\frac{5}{8} - 3\frac{1}{4} + 1\frac{2}{5}$$

$$\begin{aligned} 4\frac{5}{8} - 3\frac{1}{4} + 1\frac{2}{5} &= (4 - 3 + 1) + \left(\frac{5}{8} - \frac{1}{4} + \frac{2}{5}\right) \\ &= 2 + \frac{5 \times 5 - 10 \times 1 + 8 \times 2}{40} \\ &= 2 + \frac{25 - 10 + 16}{40} \\ &= 2 + \frac{31}{40} = 2\frac{31}{40} \end{aligned}$$

Problem 4. Find the value of $\frac{3}{7} \times \frac{14}{15}$

Dividing numerator and denominator by 3 gives:

$$\frac{1\cancel{3}}{7} \times \frac{14}{\cancel{15}_5} = \frac{1}{7} \times \frac{14}{5} = \frac{1 \times 14}{7 \times 5}$$

Dividing numerator and denominator by 7 gives:

$$\frac{1 \times \cancel{14}^2}{\cancel{7}_1 \times 5} = \frac{1 \times 2}{1 \times 5} = \frac{2}{5}$$

This process of dividing both the numerator and denominator of a fraction by the same factor(s) is called **cancelling**.

Problem 5. Evaluate $1\frac{3}{5} \times 2\frac{1}{3} \times 3\frac{3}{7}$

Mixed numbers **must** be expressed as improper fractions before multiplication can be performed. Thus,

$$\begin{aligned} 1\frac{3}{5} \times 2\frac{1}{3} \times 3\frac{3}{7} \\ &= \left(\frac{5}{5} + \frac{3}{5}\right) \times \left(\frac{6}{3} + \frac{1}{3}\right) \times \left(\frac{21}{7} + \frac{3}{7}\right) \end{aligned}$$

$$\begin{aligned} &= \frac{8}{5} \times \frac{1\cancel{7}}{\cancel{1}\cancel{7}} \times \frac{\cancel{21}^8}{\cancel{7}_1} = \frac{8 \times 1 \times 8}{5 \times 1 \times 1} \\ &= \frac{64}{5} = 12\frac{4}{5} \end{aligned}$$

Problem 6. Simplify $\frac{3}{7} \div \frac{12}{21}$

$$\frac{3}{7} \div \frac{12}{21} = \frac{3}{7}$$

Multiplying both numerator and denominator by the reciprocal of the denominator gives:

$$\frac{3}{7} \div \frac{12}{21} = \frac{1\cancel{3}}{7} \times \frac{\cancel{21}^3}{\cancel{12}_4} = \frac{3}{4} \div \frac{4}{1} = \frac{3}{4}$$

This method can be remembered by the rule: invert the second fraction and change the operation from division to multiplication. Thus:

$$\frac{3}{7} \div \frac{12}{21} = \frac{1\cancel{3}}{7} \times \frac{\cancel{21}^3}{\cancel{12}_4} = \frac{3}{4} \text{ as obtained previously.}$$

Problem 7. Find the value of $5\frac{3}{5} \div 7\frac{1}{3}$

The mixed numbers must be expressed as improper fractions. Thus,

$$5\frac{3}{5} \div 7\frac{1}{3} = \frac{28}{5} \div \frac{22}{3} = \frac{14\cancel{28}}{5} \times \frac{3}{\cancel{22}_{11}} = \frac{42}{55}$$

Problem 8. Simplify

$$\frac{1}{3} - \left(\frac{2}{5} + \frac{1}{4}\right) \div \left(\frac{3}{8} \times \frac{1}{3}\right)$$

The order of precedence of operations for problems containing fractions is the same as that for integers, i.e. remembered by **BODMAS** (Brackets, Of, Division, Multiplication, Addition and Subtraction). Thus,

$$\frac{1}{3} - \left(\frac{2}{5} + \frac{1}{4}\right) \div \left(\frac{3}{8} \times \frac{1}{3}\right)$$

$$= \frac{1}{3} - \frac{4 \times 2 + 5 \times 1}{20} \div \frac{8^1}{24_8} \quad (\text{B})$$

$$= \frac{1}{3} - \frac{13}{20} \times \frac{8^2}{1} \quad (\text{D})$$

$$= \frac{1}{3} - \frac{26}{5} \quad (\text{M})$$

$$= \frac{(5 \times 1) - (3 \times 26)}{15} \quad (\text{S})$$

$$= \frac{-73}{15} = -4\frac{13}{15}$$

Problem 9. Determine the value of

$$\frac{7}{6} \text{ of } \left(3\frac{1}{2} - 2\frac{1}{4}\right) + 5\frac{1}{8} \div \frac{3}{16} - \frac{1}{2}$$

$$\frac{7}{6} \text{ of } \left(3\frac{1}{2} - 2\frac{1}{4}\right) + 5\frac{1}{8} \div \frac{3}{16} - \frac{1}{2}$$

$$= \frac{7}{6} \text{ of } 1\frac{1}{4} + \frac{41}{8} \div \frac{3}{16} - \frac{1}{2} \quad (\text{B})$$

$$= \frac{7}{6} \times \frac{5}{4} + \frac{41}{8} \div \frac{3}{16} - \frac{1}{2} \quad (\text{O})$$

$$= \frac{7}{6} \times \frac{5}{4} + \frac{41}{8} \times \frac{16^2}{3} - \frac{1}{2} \quad (\text{D})$$

$$= \frac{35}{24} + \frac{82}{3} - \frac{1}{2} \quad (\text{M})$$

$$= \frac{35 + 656}{24} - \frac{1}{2} \quad (\text{A})$$

$$= \frac{691}{24} - \frac{1}{2} \quad (\text{A})$$

$$= \frac{691 - 12}{24} \quad (\text{S})$$

$$= \frac{679}{24} = 28\frac{7}{24}$$

Now try the following exercise

Exercise 1 Further problems on fractions

Evaluate the following:

1. (a) $\frac{1}{2} + \frac{2}{5}$ (b) $\frac{7}{16} - \frac{1}{4}$
 $\left[\text{(a) } \frac{9}{10} \quad \text{(b) } \frac{3}{16} \right]$

2. (a) $\frac{2}{7} + \frac{3}{11}$ (b) $\frac{2}{9} - \frac{1}{7} + \frac{2}{3}$
 $\left[\text{(a) } \frac{43}{77} \quad \text{(b) } \frac{47}{63} \right]$

3. (a) $10\frac{3}{7} - 8\frac{2}{3}$ (b) $3\frac{1}{4} - 4\frac{4}{5} + 1\frac{5}{6}$
 $\left[\text{(a) } 1\frac{16}{21} \quad \text{(b) } \frac{17}{60} \right]$

4. (a) $\frac{3}{4} \times \frac{5}{9}$ (b) $\frac{17}{35} \times \frac{15}{119}$
 $\left[\text{(a) } \frac{5}{12} \quad \text{(b) } \frac{3}{49} \right]$

5. (a) $\frac{3}{5} \times \frac{7}{9} \times 1\frac{2}{7}$ (b) $\frac{13}{17} \times 4\frac{7}{11} \times 3\frac{4}{39}$
 $\left[\text{(a) } \frac{3}{5} \quad \text{(b) } 11 \right]$

6. (a) $\frac{3}{8} \div \frac{45}{64}$ (b) $1\frac{1}{3} \div 2\frac{5}{9}$
 $\left[\text{(a) } \frac{8}{15} \quad \text{(b) } \frac{12}{23} \right]$

7. $\frac{1}{2} + \frac{3}{5} \div \frac{8}{15} - \frac{1}{3}$ $\left[1\frac{7}{24} \right]$

8. $\frac{7}{15} \text{ of } \left(15 \times \frac{5}{7}\right) + \left(\frac{3}{4} \div \frac{15}{16}\right)$ $\left[5\frac{4}{5} \right]$

9. $\frac{1}{4} \times \frac{2}{3} - \frac{1}{3} \div \frac{3}{5} + \frac{2}{7}$ $\left[-\frac{13}{126} \right]$

10. $\left(\frac{2}{3} \times 1\frac{1}{4}\right) \div \left(\frac{2}{3} + \frac{1}{4}\right) + 1\frac{3}{5}$ $\left[2\frac{28}{55} \right]$

1.2 Ratio and proportion

The ratio of one quantity to another is a fraction, and is the number of times one quantity is contained in another quantity **of the same kind**. If one quantity is **directly proportional** to another, then as one quantity doubles, the other quantity also doubles. When a quantity is **inversely proportional** to another, then as one quantity doubles, the other quantity is halved.

Problem 10. A piece of timber 273 cm long is cut into three pieces in the ratio of 3 to 7 to 11. Determine the lengths of the three pieces

The total number of parts is $3 + 7 + 11$, that is, 21.
Hence 21 parts correspond to 273 cm

1 part corresponds to $\frac{273}{21} = 13$ cm

3 parts correspond to $3 \times 13 = 39$ cm

7 parts correspond to $7 \times 13 = 91$ cm

11 parts correspond to $11 \times 13 = 143$ cm

i.e. **the lengths of the three pieces are 39 cm, 91 cm and 143 cm.**

(Check: $39 + 91 + 143 = 273$)

Problem 11. A gear wheel having 80 teeth is in mesh with a 25 tooth gear. What is the gear ratio?

$$\text{Gear ratio} = 80:25 = \frac{80}{25} = \frac{16}{5} = 3.2$$

i.e. gear ratio = **16 : 5** or **3.2 : 1**

Problem 12. An alloy is made up of metals A and B in the ratio 2.5 : 1 by mass. How much of A has to be added to 6 kg of B to make the alloy?

Ratio A : B : 2.5 : 1 (i.e. A is to B as 2.5 is to 1)
or $\frac{A}{B} = \frac{2.5}{1} = 2.5$

When $B = 6$ kg, $\frac{A}{6} = 2.5$ from which,

$$A = 6 \times 2.5 = \mathbf{15 \text{ kg}}$$

Problem 13. If 3 people can complete a task in 4 hours, how long will it take 5 people to complete the same task, assuming the rate of work remains constant

The more the number of people, the more quickly the task is done, hence inverse proportion exists.

3 people complete the task in 4 hours,

1 person takes three times as long, i.e.

$$4 \times 3 = 12 \text{ hours,}$$

5 people can do it in one fifth of the time that one person takes, that is $\frac{12}{5}$ hours or **2 hours 24 minutes.**

Now try the following exercise

Exercise 5 Further problems on ratio and proportion

- Divide 621 cm in the ratio of 3 to 7 to 13.
[81 cm to 189 cm to 351 cm]
- When mixing a quantity of paints, dyes of four different colours are used in the ratio of 7:3:19:5. If the mass of the first dye used is $3\frac{1}{2}$ g, determine the total mass of the dyes used. [17 g]
- Determine how much copper and how much zinc is needed to make a 99 kg brass ingot if they have to be in the proportions copper : zinc : 8 : 3 by mass.
[72 kg : 27 kg]
- It takes 21 hours for 12 men to resurface a stretch of road. Find how many men it takes to resurface a similar stretch of road in 50 hours 24 minutes, assuming the work rate remains constant. [5]
- It takes 3 hours 15 minutes to fly from city A to city B at a constant speed. Find how long the journey takes if
 - the speed is $1\frac{1}{2}$ times that of the original speed and
 - if the speed is three-quarters of the original speed.
 [(a) 2 h 10 min (b) 4 h 20 min]

1.3 Decimals

The decimal system of numbers is based on the **digits** 0 to 9. A number such as 53.17 is called a **decimal fraction**, a decimal point separating the integer part, i.e. 53, from the fractional part, i.e. 0.17

A number which can be expressed exactly as a decimal fraction is called a **terminating decimal** and those which cannot be expressed exactly as a decimal fraction are called **non-terminating decimals**. Thus, $\frac{3}{2} = 1.5$ is a terminating decimal, but $\frac{4}{3} = 1.33333\dots$ is a non-terminating decimal. $1.33333\dots$ can be written as $1.\dot{3}$, called 'one point-three recurring'.

The answer to a non-terminating decimal may be expressed in two ways, depending on the accuracy required:

- (i) correct to a number of **significant figures**, that is, figures which signify something, and
- (ii) correct to a number of **decimal places**, that is, the number of figures after the decimal point.

The last digit in the answer is unaltered if the next digit on the right is in the group of numbers 0, 1, 2, 3 or 4, but is increased by 1 if the next digit on the right is in the group of numbers 5, 6, 7, 8 or 9. Thus the non-terminating decimal $7.6183\dots$ becomes 7.62 , correct to 3 significant figures, since the next digit on the right is 8, which is in the group of numbers 5, 6, 7, 8 or 9. Also $7.6183\dots$ becomes 7.618 , correct to 3 decimal places, since the next digit on the right is 3, which is in the group of numbers 0, 1, 2, 3 or 4.

Problem 14. Evaluate
 $42.7 + 3.04 + 8.7 + 0.06$

The numbers are written so that the decimal points are under each other. Each column is added, starting from the right.

$$\begin{array}{r} 42.7 \\ 3.04 \\ 8.7 \\ 0.06 \\ \hline 54.50 \end{array}$$

Thus $42.7 + 3.04 + 8.7 + 0.06 = 54.50$

Problem 15. Take 81.70 from 87.23

The numbers are written with the decimal points under each other.

$$\begin{array}{r} 87.23 \\ -81.70 \\ \hline 5.53 \end{array}$$

Thus $87.23 - 81.70 = 5.53$

Problem 16. Find the value of
 $23.4 - 17.83 - 57.6 + 32.68$

The sum of the positive decimal fractions is

$$23.4 + 32.68 = 56.08$$

The sum of the negative decimal fractions is

$$17.83 + 57.6 = 75.43$$

Taking the sum of the negative decimal fractions from the sum of the positive decimal fractions gives:

$$56.08 - 75.43$$

i.e. $-(75.43 - 56.08) = -19.35$

Problem 17. Determine the value of
 74.3×3.8

When multiplying decimal fractions: (i) the numbers are multiplied as if they are integers, and (ii) the position of the decimal point in the answer is such that there are as many digits to the right of it as the sum of the digits to the right of the decimal points of the two numbers being multiplied together. Thus

$$\begin{array}{r} (i) \quad 743 \\ \quad 38 \\ \hline 5944 \\ 22290 \\ \hline 28234 \end{array}$$

(ii) As there are $(1 + 1) = 2$ digits to the right of the decimal points of the two numbers being multiplied together, ($74.\underline{3} \times 3.\underline{8}$), then

$$74.3 \times 3.8 = 282.34$$

Problem 18. Evaluate $37.81 \div 1.7$, correct to (i) 4 significant figures and (ii) 4 decimal places

$$37.81 \div 1.7 = \frac{37.81}{1.7}$$

The denominator is changed into an integer by multiplying by 10. The numerator is also multiplied by 10 to keep the fraction the same. Thus

$$37.81 \div 1.7 = \frac{37.81 \times 10}{1.7 \times 10} = \frac{378.1}{17}$$

The long division is similar to the long division of integers and the first four steps are as shown:

$$\begin{array}{r} 22.24117.. \\ 17 \overline{) 378.100000} \\ \underline{34} \\ 38 \\ \underline{34} \\ 41 \\ \underline{34} \\ 70 \\ \underline{68} \\ 20 \end{array}$$

- (i) $37.81 \div 1.7 = 22.24$, correct to 4 significant figures, and
- (ii) $37.81 \div 1.7 = 22.2412$, correct to 4 decimal places.

Problem 19. Convert (a) 0.4375 to a proper fraction and (b) 4.285 to a mixed number

- (a) 0.4375 can be written as $\frac{0.4375 \times 10\,000}{10\,000}$ without changing its value,

$$\text{i.e. } 0.4375 = \frac{4375}{10\,000}$$

By cancelling

$$\frac{4375}{10\,000} = \frac{875}{2000} = \frac{175}{400} = \frac{35}{80} = \frac{7}{16}$$

$$\text{i.e. } \mathbf{0.4375 = \frac{7}{16}}$$

- (b) Similarly, $4.285 = 4\frac{285}{1000} = 4\frac{57}{200}$

Problem 20. Express as decimal fractions:

(a) $\frac{9}{16}$ and (b) $5\frac{7}{8}$

- (a) To convert a proper fraction to a decimal fraction, the numerator is divided by the denominator. Division by 16 can be done by the long division method, or, more simply, by dividing by 2 and then 8:

$$\begin{array}{r} 4.50 \\ 2 \overline{) 9.00} \\ 8 \overline{) 4.5000} \end{array}$$

$$\text{Thus, } \frac{9}{16} = \mathbf{0.5625}$$

- (b) For mixed numbers, it is only necessary to convert the proper fraction part of the mixed number to a decimal fraction. Thus, dealing with the $\frac{7}{8}$ gives:

$$\begin{array}{r} 0.875 \\ 8 \overline{) 7.000} \end{array} \quad \text{i.e. } \frac{7}{8} = 0.875$$

$$\text{Thus } \mathbf{5\frac{7}{8} = 5.875}$$

Now try the following exercise

Exercise 3 Further problems on decimals

In Problems 1 to 6, determine the values of the expressions given:

1. $23.6 + 14.71 - 18.9 - 7.421$ [11.989]
2. $73.84 - 113.247 + 8.21 - 0.068$ [-31.265]
3. $3.8 \times 4.1 \times 0.7$ [10.906]
4. 374.1×0.006 [2.2446]
5. $421.8 \div 17$, (a) correct to 4 significant figures and (b) correct to 3 decimal places. [(a) 24.81 (b) 24.812]
6. $\frac{0.0147}{2.3}$, (a) correct to 5 decimal places and (b) correct to 2 significant figures. [(a) 0.00639 (b) 0.0064]

7. Convert to proper fractions:

(a) 0.65 (b) 0.84 (c) 0.0125 (d) 0.282
and (e) 0.024

$\left[\begin{array}{l} \text{(a) } \frac{13}{20} \quad \text{(b) } \frac{21}{25} \quad \text{(c) } \frac{1}{80} \quad \text{(d) } \frac{141}{500} \quad \text{(e) } \frac{3}{125} \end{array} \right]$

8. Convert to mixed numbers:

(a) 1.82 (b) 4.275 (c) 14.125 (d) 15.35
and (e) 16.2125

$\left[\begin{array}{l} \text{(a) } 1\frac{41}{50} \quad \text{(b) } 4\frac{11}{40} \quad \text{(c) } 14\frac{1}{8} \\ \text{(d) } 15\frac{7}{20} \quad \text{(e) } 16\frac{17}{80} \end{array} \right]$

In Problems 9 to 12, express as decimal fractions to the accuracy stated:

9. $\frac{4}{9}$, correct to 5 significant figures.
[0.44444]

10. $\frac{17}{27}$, correct to 5 decimal places.
[0.62963]

11. $1\frac{9}{16}$, correct to 4 significant figures.
[1.563]

12. $13\frac{31}{37}$, correct to 2 decimal places.
[13.84]

1.4 Percentages

Percentages are used to give a common standard and are fractions having the number 100 as their denominators. For example, 25 per cent means $\frac{25}{100}$ i.e. $\frac{1}{4}$ and is written 25%.

Problem 21. Express as percentages:
(a) 1.875 and (b) 0.0125

A decimal fraction is converted to a percentage by multiplying by 100. Thus,

(a) 1.875 corresponds to $1.875 \times 100\%$, i.e. **187.5%**

(b) 0.0125 corresponds to $0.0125 \times 100\%$, i.e. **1.25%**

Problem 22. Express as percentages:

(a) $\frac{5}{16}$ and (b) $1\frac{2}{5}$

To convert fractions to percentages, they are (i) converted to decimal fractions and (ii) multiplied by 100

(a) By division, $\frac{5}{16} = 0.3125$, hence $\frac{5}{16}$ corresponds to $0.3125 \times 100\%$, i.e. **31.25%**

(b) Similarly, $1\frac{2}{5} = 1.4$ when expressed as a decimal fraction.

Hence $1\frac{2}{5} = 1.4 \times 100\% = \mathbf{140\%}$

Problem 23. It takes 50 minutes to machine a certain part. Using a new type of tool, the time can be reduced by 15%. Calculate the new time taken

$$\begin{aligned} 15\% \text{ of } 50 \text{ minutes} &= \frac{15}{100} \times 50 = \frac{750}{100} \\ &= 7.5 \text{ minutes.} \end{aligned}$$

hence the **new time taken** is

$$50 - 7.5 = \mathbf{42.5 \text{ minutes.}}$$

Alternatively, if the time is reduced by 15%, then it now takes 85% of the original time, i.e. 85% of

$$50 = \frac{85}{100} \times 50 = \frac{4250}{100} = \mathbf{42.5 \text{ minutes}}, \text{ as above.}$$

Problem 24. Find 12.5% of £378

12.5% of £378 means $\frac{12.5}{100} \times 378$, since per cent means 'per hundred'.

$$\begin{aligned} \text{Hence } 12.5\% \text{ of } \pounds 378 &= \frac{12.5}{100} \times 378 = \frac{1}{8} \times 378 = \\ &= \frac{378}{8} = \mathbf{\pounds 47.25} \end{aligned}$$

Problem 25. Express 25 minutes as a percentage of 2 hours, correct to the nearest 1%

Working in minute units, 2 hours = 120 minutes.
Hence 25 minutes is $\frac{25}{120}$ ths of 2 hours. By cancelling, $\frac{25}{120} = \frac{5}{24}$

Expressing $\frac{5}{24}$ as a decimal fraction gives 0.208 $\bar{3}$

Multiplying by 100 to convert the decimal fraction to a percentage gives:

$$0.208\bar{3} \times 100 = 20.8\bar{3}\%$$

Thus **25 minutes is 21% of 2 hours**, correct to the nearest 1%.

Problem 26. A German silver alloy consists of 60% copper, 25% zinc and 15% nickel. Determine the masses of the copper, zinc and nickel in a 3.74 kilogram block of the alloy

By direct proportion:

100% corresponds to 3.74 kg

$$1\% \text{ corresponds to } \frac{3.74}{100} = 0.0374 \text{ kg}$$

60% corresponds to $60 \times 0.0374 = 2.244$ kg

25% corresponds to $25 \times 0.0374 = 0.935$ kg

15% corresponds to $15 \times 0.0374 = 0.561$ kg

Thus, the masses of the copper, zinc and nickel are **2.244 kg, 0.935 kg and 0.561 kg**, respectively.

(Check: $2.244 + 0.935 + 0.561 = 3.74$)

Now try the following exercise

Exercise 4 Further problems percentages

1. Convert to percentages:

(a) 0.057 (b) 0.374 (c) 1.285

[(a) 5.7% (b) 37.4% (c) 128.5%]

2. Express as percentages, correct to 3 significant figures:

(a) $\frac{7}{33}$ (b) $\frac{19}{24}$ (c) $1\frac{11}{16}$

[(a) 21.2% (b) 79.2% (c) 169%]

3. Calculate correct to 4 significant figures:

(a) 18% of 2758 tonnes (b) 47% of 18.42 grams (c) 147% of 14.1 seconds

[(a) 496.4 t (b) 8.657 g (c) 20.73 s]

4. When 1600 bolts are manufactured, 36 are unsatisfactory. Determine the percentage unsatisfactory. [2.25%]

5. Express: (a) 140 kg as a percentage of 1 t (b) 47 s as a percentage of 5 min (c) 13.4 cm as a percentage of 2.5 m

[(a) 14% (b) 15.67% (c) 5.36%]

6. A block of monel alloy consists of 70% nickel and 30% copper. If it contains 88.2 g of nickel, determine the mass of copper in the block. [37.8 g]

7. A drilling machine should be set to 250 rev/min. The nearest speed available on the machine is 268 rev/min. Calculate the percentage over speed. [7.2%]

8. Two kilograms of a compound contains 30% of element A, 45% of element B and 25% of element C. Determine the masses of the three elements present.

[A 0.6 kg, B 0.9 kg, C 0.5 kg]

9. A concrete mixture contains seven parts by volume of ballast, four parts by volume of sand and two parts by volume of cement. Determine the percentage of each of these three constituents correct to the nearest 1% and the mass of cement in a two tonne dry mix, correct to 1 significant figure.

[54%, 31%, 15%, 0.3 t]