1

Basic arithmetic

1.1 Arithmetic operations

Whole numbers are called **integers**. $+3, +5, +72$ are called positive integers; $-13, -6, -51$ are called negative integers. Between positive and negative integers is the number 0 which is neither positive nor negative.

The four basic arithmetic operators are: add $(+)$, subtract $(-)$, multiply $(\times)$ and divide $(\div)$

For addition and subtraction, when **unlike signs** are together in a calculation, the overall sign is **negative**. Thus, adding minus 4 to 3 is $3 + -4$ and becomes $3 - 4 = -1$. **Like signs** together give an overall **positive sign**. Thus subtracting minus 4 from 3 is $3 - -4$ and becomes $3 + 4 = 7$. For multiplication and division, when the numbers have **unlike signs**, the answer is **negative**, but when the numbers have **like signs** the answer is **positive**. Thus $3 \times -4 = -12$, whereas $-3 \times -4 = +12$. Similarly

$$\frac{4}{-3} = -\frac{4}{3} \quad \text{and} \quad \frac{-4}{-3} = +\frac{4}{3}$$

Problem 1. Add 27, −74, 81 and −19

This problem is written as $27 - 74 + 81 - 19$

Adding the positive integers:	27
	81
Sum of positive integers is:	108
Adding the negative integers:	74
	19
Sum of negative integers is:	93

Taking the sum of the negative integers from the sum of the positive integers gives:

$$\begin{array}{r} 108 \\ -93 \\ \hline 15 \end{array}$$

Thus $\mathbf{27 - 74 + 81 - 19 = 15}$

Problem 2. Subtract 89 from 123

This is written mathematically as $123 - 89$

$$\begin{array}{r} 123 \\ -89 \\ \hline 34 \end{array}$$

Thus $\mathbf{123 - 89 = 34}$

Problem 3. Subtract −74 from 377

This problem is written as $377 - \ - 74$. Like signs together give an overall positive sign, hence

$377 - -74 = 377 + 74$

$$\begin{array}{r} 377 \\ +74 \\ \hline 451 \end{array}$$

Thus $\mathbf{377 - -74 = 451}$

Problem 4. Subtract 243 from 126

The problem is $126 - 243$. When the second number is larger than the first, take the smaller number from the larger and make the result negative.

Thus $126 - 243 = -(243 - 126)$

$$\begin{array}{r} 243 \\ -126 \\ \hline 117 \end{array}$$

Thus $\mathbf{126 - 243 = -117}$

Problem 5. Subtract 318 from −269

$-269 - 318$. The sum of the negative integers is

$$\begin{array}{r} 269 \\ +318 \\ \hline 587 \end{array}$$

Thus $\mathbf{-269 - 318 = -587}$

Problem 6. Multiply 74 by 13

This is written as 74×13

$$\begin{array}{rrl} & 74 & \\ & 13 & \\ \hline & 222 & \leftarrow 74 \times 3 \\ & 740 & \leftarrow 74 \times 10 \\ \text{Adding:} & 962 & \end{array}$$

Thus $\mathbf{74 \times 13 = 962}$

Problem 7. Multiply by 178 by -46

When the numbers have different signs, the result will be negative. (With this in mind, the problem can now be solved by multiplying 178 by 46)

$$\begin{array}{r} 178 \\ 46 \\ \hline 1068 \\ 7120 \\ \hline 8188 \end{array}$$

Thus $178 \times 46 = 8188$ and $\mathbf{178 \times (-46) = -8188}$

Problem 8. Divide l043 by 7

When dividing by the numbers 1 to 12, it is usual to use a method called **short division**.

$$7\overline{)\,10^{3}4^{6}3}\quad 1\,4\,9$$

Step 1. 7 into 10 goes 1, remainder 3. Put 1 above the 0 of 1043 and carry the 3 remainder to the next digit on the right, making it 34;

Step 2. 7 into 34 goes 4, remainder 6. Put 4 above the 4 of 1043 and carry the 6 remainder to the next digit on the right, making it 63;

Step 3. 7 into 63 goes 9, remainder 0. Put 9 above the 3 of 1043.

Thus $\mathbf{1043 \div 7 = 149}$

Problem 9. Divide 378 by 14

When dividing by numbers which are larger than 12, it is usual to use a method called **long division**.

$$\begin{array}{rrl} & 27 & \\ 14\overline{)} & 378 & \\ (2)\ 2 \times 14 \rightarrow & 28 & \\ \hline & 98 & \\ (4)\ 7 \times 14 \rightarrow & 98 & \\ \hline & 00 & \end{array}$$

(1) 14 into 37 goes twice. Put 2 above the 7 of 378.
(3) Subtract. Bring down the 8. 14 into 98 goes 7 times. Put 7 above the 8 of 378.
(5) Subtract.

Thus $\mathbf{378 \div 14 = 27}$

Problem 10. Divide 5669 by 46

This problem may be written as $\frac{5669}{46}$ or $5669 \div 46$ or 5669/46

Using the long division method shown in Problem 9 gives:

$$\begin{array}{rl} & 123 \\ 46\overline{)} & 5669 \\ & 46 \\ \hline & 106 \\ & 92 \\ \hline & 149 \\ & 138 \\ \hline & 11 \end{array}$$

As there are no more digits to bring down,

$$\mathbf{5669 \div 46 = 123,\ remainder\ 11} \quad \text{or} \quad \mathbf{123\frac{11}{46}}$$

Now try the following exercise

Exercise 1 Further problems on arithmetic operations (Answers on page 269)

In Problems 1 to 24, determine the values of the expressions given:

1. $67 - 82 + 34$
2. $124 - 273 + 481 - 398$
3. $927 - 114 + 182 - 183 - 247$
4. $2417 - 487 + 2424 - 1778 - 4712$
5. $-38419 - 2177 + 2440 - 799 + 2834$
6. $2715 - 18250 + 11471 - 1509 + 113274$
7. $73 - 57$
8. $813 - (-674)$
9. $647 - 872$
10. $3151 - (-2763)$
11. $4872 - 4683$

12. $-23148 - 47724$
13. $38441 - 53774$
14. (a) 261×7 (b) 462×9
15. (a) 783×11 (b) 73×24
16. (a) 27×38 (b) 77×29
17. (a) 448×23 (b) $143 \times (-31)$
18. (a) $288 \div 6$ (b) $979 \div 11$
19. (a) $\frac{1813}{7}$ (b) $\frac{896}{16}$
20. (a) $\frac{21432}{47}$ (b) $15904 \div 56$
21. (a) $\frac{88738}{187}$ (b) $46857 \div 79$
22. A screw has a mass of 15 grams. Calculate, in kilograms, the mass of 1200 such screws.
23. Holes are drilled 35.7 mm apart in a metal plate. If a row of 26 holes is drilled, determine the distance, in centimetres, between the centres of the first and last holes.
24. Calculate the diameter d and dimensions A and B for the template shown in Figure 1.1. All dimensions are in millimetres.

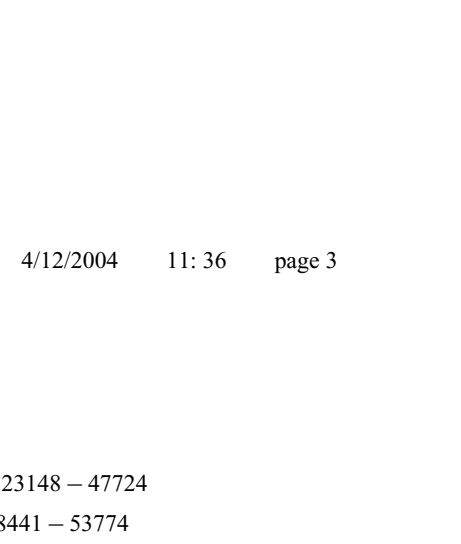

Fig. 1.1

1.2 Highest common factors and lowest common multiples

When two or more numbers are multiplied together, the individual numbers are called **factors**. Thus a factor is a number which divides into another number exactly. The **highest common factor (HCF)** is the largest number which divides into two or more numbers exactly.

A **multiple** is a number which contains another number an exact number of times. The smallest number which is exactly divisible by each of two or more numbers is called the **lowest common multiple (LCM)**.

Problem 11. Determine the HCF of the numbers 12, 30 and 42

Each number is expressed in terms of its lowest factors. This is achieved by repeatedly dividing by the prime numbers 2, 3, 5, 7, 11, 13 . . . (where possible) in turn. Thus

$$12 = 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \times 7$$

The factors which are common to each of the numbers are 2 in column 1 and 3 in column 3, shown by the broken lines. Hence the **HCF is 2×3, i.e. 6**. That is, 6 is the largest number which will divide into 12, 30 and 42.

Problem 12. Determine the HCF of the numbers 30, 105, 210 and 1155

Using the method shown in Problem 11:

$$30 = 2 \times 3 \times 5$$
$$105 = 3 \times 5 \times 7$$
$$210 = 2 \times 3 \times 5 \times 7$$
$$1155 = 3 \times 5 \times 7 \times 11$$

The factors which are common to each of the numbers are 3 in column 2 and 5 in column 3. Hence **the HCF is $3 \times 5 = 15$**

Problem 13. Determine the LCM of the numbers 12, 42 and 90

The LCM is obtained by finding the lowest factors of each of the numbers, as shown in Problems 11 and 12 above, and then selecting the largest group of any of the factors present. Thus

$$12 = 2 \times 2 \times 3$$
$$42 = 2 \times 3 \times 7$$
$$90 = 2 \times 3 \times 3 \times 5$$

The largest group of any of the factors present are shown by the broken lines and are 2×2 in 12, 3×3 in 90, 5 in 90 and 7 in 42.

Hence **the LCM is 2 x 2 x 3 x 3 x 5 x 7 = 1260**, and is the smallest number which 12, 42 and 90 will all divide into exactly.

Problem 14. Determine the LCM of the numbers 150, 210, 735 and 1365

Using the method shown in Problem 13 above:

$$150 = \boxed{2} \times \boxed{3} \times \boxed{5 \times 5}$$
$$210 = 2 \times 3 \times 5 \times 7$$
$$735 = 3 \times 5 \times \boxed{7 \times 7}$$
$$1365 = 3 \times 5 \times 7 \times \boxed{13}$$

The LCM is 2 x 3 x 5 x 5 x 7 x 7 x 13 = 95550

Now try the following exercise

Exercise 2 Further problems on highest common factors and lowest common multiples (Answers on page 269)

In Problems 1 to 6 find (a) the HCF and (b) the LCM of the numbers given:

1. 6, 10, 14
2. 12, 30, 45
3. 10, 15, 70, 105
4. 90, 105, 300
5. 196, 210, 910, 462
6. 196, 350, 770

1.3 Order of precedence and brackets

When a particular arithmetic operation is to be performed first, the numbers and the operator(s) are placed in brackets. Thus 3 times the result of 6 minus 2 is written as $3 \times (6 - 2)$. In arithmetic operations, the order in which operations are performed are:

(i) to determine the values of operations contained in brackets;

(ii) multiplication and division (the word 'of' also means multiply); and

(iii) addition and subtraction.

This **order of precedence** can be remembered by the word **BODMAS**, standing for **B**rackets, **O**f, **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction, taken in that order.

The basic laws governing the use of brackets and operators are shown by the following examples:

(i) $2 + 3 = 3 + 2$, i.e. the order of numbers when adding does not matter;

(ii) $2 \times 3 = 3 \times 2$, i.e. the order of numbers when multiplying does not matter;

(iii) $2 + (3 + 4) = (2 + 3) + 4$, i.e. the use of brackets when adding does not affect the result;

(iv) $2 \times (3 \times 4) = (2 \times 3) \times 4$, i.e. the use of brackets when multiplying does not affect the result;

(v) $2 \times (3 + 4) = 2(3 + 4) = 2 \times 3 + 2 \times 4$, i.e. a number placed outside of a bracket indicates that the whole contents of the bracket must be multiplied by that number;

(vi) $(2 + 3)(4 + 5) = (5)(9) = 45$, i.e. adjacent brackets indicate multiplication;

(vii) $2[3 + (4 \times 5)] = 2[3 + 20] = 2 \times 23 = 46$, i.e. when an expression contains inner and outer brackets, the inner brackets are removed first.

Problem 15. Find the value of $6 + 4 \div (5 - 3)$

The order of precedence of operations is remembered by the word BODMAS.

Thus $6 + 4 \div (5 - 3) = 6 + 4 \div 2$ (Brackets)
$= 6 + 2$ (Division)
$= \mathbf{8}$ (Addition)

Problem 16. Determine the value of

$$13 - 2 \times 3 + 14 \div (2 + 5)$$

$13 - 2 \times 3 + 14 \div (2 + 5) = 13 - 2 \times 3 + 14 \div 7$ (B)
$= 13 - 2 \times 3 + 2$ (D)
$= 13 - 6 + 2$ (M)
$= 15 - 6$ (A)
$= \mathbf{9}$ (S)

Problem 17. Evaluate

$$16 \div (2 + 6) + 18[3 + (4 \times 6) - 21]$$

$16 \div (2 + 6) + 18[3 + (4 \times 6) - 21]$
$= 16 \div (2 + 6) + 18[3 + 24 - 21)$ (B)
$= 16 \div 8 + 18 \times 6$ (B)
$= 2 + 18 \times 6$ (D)
$= 2 + 108$ (M)
$= \mathbf{110}$ (A)

Problem 18. Find the value of

$$23 - 4(2 \times 7) + \frac{(144 \div 4)}{(14 - 8)}$$

$$\begin{aligned} 23 - 4(2 \times 7) + \frac{(144 \div 4)}{(14 - 8)} &= 23 - 4 \times 14 + \frac{36}{6} && \text{(B)} \\ &= 23 - 4 \times 14 + 6 && \text{(D)} \\ &= 23 - 56 + 6 && \text{(M)} \\ &= 29 - 56 && \text{(A)} \\ &= \mathbf{-27} && \text{(S)} \end{aligned}$$

Now try the following exercise

Exercise 3 Further problems on order of precedence and brackets (Answers on page 269)

Simplify the expressions given in Problems 1 to 7:

1. $14 + 3 \times 15$
2. $17 - 12 \div 4$
3. $86 + 24 \div (14 - 2)$
4. $7(23 - 18) \div (12 - 5)$
5. $63 - 28(14 \div 2) + 26$
6. $\frac{112}{16} - 119 \div 17 + (3 \times 19)$
7. $\frac{(50 - 14)}{3} + 7(16 - 7) - 7$